A New Design of Microwave Filters by Using Continuously Varying Transmission lines

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Abstract

This paper outlines a new original method for microwave filter design. A cubic spline interpolation is used to design filter with a continuously changing profile. Then, the scattering parameters of a non-uniform transmission line with a cubic polynomial variation of its characteristic impedance are studied in detail in frequency domain. In order to validate this method, a wideband bandstop filter consisting in a non-uniform microstrip line with continuously varying width was optimized, designed, constructed and measured.

I - Introduction

Non-uniform transmission lines are usually used as impedance matching devices, pulse transformers, resonators, directionnal couplers and filters. The theory for exponential [1], [2], parabolic [3], [4], cosine-squared [4] and hyperbolic transmission lines supporting a TEM or nearly TEM mode is well established in frequency and time domain. For all these structures the common method of analysis is based on the equivalent TEM transmission line model. Recently, P.P. Roberts and G.E. Town [5] proposed an original method using the theory of inverse scattering to design microwave filters with smooth variation of their profile. Filters designed by this method show distinct advantages over other widespread techniques. But this method is complicated and requires numerical techniques which can not avoid truncation errors. Moreover, a lack of generality prevents it from being applied to analyze different kinds of non-uniform transmission lines which could be used in more varied applications than filter designs. This paper presents a generalized theory for designing an optimum non-uniform line. This line can be optimized to perform a filter, but moreover its optimization can have another aim, for example designing an impedance matching device.

First, the global variation of the line is described by a set of discrete points. The continuous shape of the variation is then ensured by cubic spline interpolation. Between two consecutive discrete points, the variation of the characteristic impedance follows a cubic polynomial. The frequency-domain scattering parameters of a lossless transmission line with such a variation is presented. The performances of the filter are improved by optimizing the shape of the line.

II - Global design of the line shape

The first step consists in describing the characteristic impedance variation of the line. A cubic spline interpolation was chosen for two reasons. First, it allows to create a curve passing through a set of discrete points, where ondulations between two points are minimized. Secondly, cubic spline expression is general sufficient to describe or make approximation of a large class of functions, variations or ondulations. Cubic spline interpolation can be applied to regular or irregular spaces between two discrete points. It also ensures the continuity of the first and second derivatives. The impedance variation between two discrete points is described by a cubic polynomial where x varies from 0 to λ :

$$Z(x) = Z_0 + Z_1\,x + Z_2\,x^2 + Z_3\,x^3 \qquad (1)$$

with $Z_3 = \dfrac{-Z''_j + Z''_{j+1}}{6\lambda}$, $\quad Z_2 = \dfrac{Z''_j}{2}$,

$$Z_1 = -\frac{\lambda}{3}Z''_j - \frac{\lambda}{6}Z''_{j+1} - \frac{Zd_j - Zd_{j+1}}{\lambda},$$

and $Z_0 = Zd_j$

Zd_j represents the ordinate of the j^{th} discrete point used to define the cubic spline interpolation, and Z''_j is the second derivative of the impedance variation Z(x) at this point.

III - Scattering parameters in frequency domain

III-1 Cubic polynomial transmission line

A non-uniform transmission line with a cubic polynomial variation of its characteristic impedance can be described as a set of scattering parameters. These scattering parameters connect two incident waves $a_1(\omega)$, $a_2(\omega)$ and two reflected waves $b_1(\omega)$ and $b_2(\omega)$, as illustrated in Fig. 1.

$$b_1 = S_{11}\cdot a_1 + S_{12}\cdot a_2 \qquad (2)$$

$$b_2 = S_{21}\cdot a_1 + S_{22}\cdot a_2 \qquad (3)$$

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A lossless non-uniform line is considered. It extends from $x = 0$ to $x = \lambda$; this line is terminated with two different reference lines at both ends. These lines, Z_{ref1} and Z_{ref2}, have for characteristic impedance the impedance of the cubic line at the left and right hand sides, respectively.

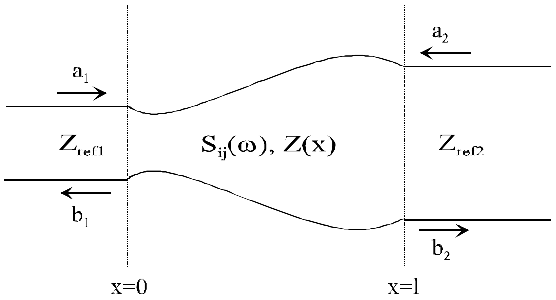

Fig. 1 - Scattering parameters description of a cubic transmission line.

The cubic line has a characteristic impedance varying as in equation (1), with the same coefficients. This relation can also be expressed in the form of:

$$Z(x) = \sqrt{\frac{L(x)}{C(x)}} \tag{4}$$

with $$L(x) = \sqrt{L_0 \cdot C_0} \cdot (Z_0 + Z_1 \cdot x + Z_2 \cdot x^2 + Z_3 \cdot x^3) \tag{5}$$

and $$C(x) = \frac{\sqrt{L_0 \cdot C_0}}{Z_0 + Z_1 \cdot x + Z_2 \cdot x^2 + Z_3 \cdot x^3} \tag{6}$$

At the input end of the non-uniform transmission line, $Z_0 = \sqrt{\frac{L_0}{C_0}}$, where L_0 and C_0 are the inductance and capacitance at $x = 0$. Using the TEM transmission line model, and equations (4), (5), (6) the frequency domain voltage V(x) and current I(x) along this line are defined as:

$$\begin{cases} \frac{\partial^2 V}{\partial x^2} - \frac{1}{Z(x)} \cdot \frac{\partial Z(x)}{\partial x} \cdot \frac{\partial V}{\partial x} + L \cdot C \cdot \omega^2 \cdot V = 0 \\ \frac{\partial^2 I}{\partial x^2} + \frac{1}{Z(x)} \cdot \frac{\partial Z(x)}{\partial x} \cdot \frac{\partial I}{\partial x} + L \cdot C \cdot \omega^2 \cdot I = 0 \end{cases} \tag{7}$$

The solution of the voltage equation is written as follows:

$$V(x) = a(x).p + b(x).q \tag{8}$$

where p and q are two constants and a(x), b(x) represent two expansions in power series:

$$a(x) = \sum_{j=0}^{\infty} a_j \cdot x^j \tag{9}$$

and $$b(x) = \sum_{j=0}^{\infty} b_j \cdot x^{j+1} \tag{10}$$

The current relation can be expressed by means of the two solutions of the voltage relation (8).

$$I(x) = \frac{j \cdot v}{Z(x) \cdot \omega} (p.ap(x) + q.bp(x)) \tag{11}$$

where ap(x) and bp(x) are the first derivatives of a(x) and b(x), respectively. v is the propagation speed through the line. The convergence of these four developpements is ensured by the Fuchs theorem. By including (9) and (10) into (8) and by equating terms of same degrees, two recurrence relations of order five are obtained.

p and q in relation (8) and (11) are determined **by the boundary conditions** as in [1] and [2].

** at the output end, we have:*

$$\left.\frac{V(x)}{I(x)}\right|_{x=\lambda} = Z_L \tag{12}$$

where $Z_L = Z(x=\lambda) = Z_0 + Z_1\lambda + Z_2\lambda^2 + Z_3\lambda^3$

From this relation, p/q is expressed as follows :

$$\frac{p}{q} = -\frac{j \cdot b + \frac{v}{\omega} bp}{j \cdot a + \frac{v}{\omega} ap} \tag{13}$$

with $a = a(x = \lambda)$, and similarly for b, ap and bp.

** at the input end, we have:*

$$\left.\frac{V(x)}{I(x)}\right|_{x=0} = Z_{in} \tag{14}$$

where Z_{in} is the input impedance, and can be expressed from the ratio p/q. ρ is defined as the reflection coefficient at the input end :

$$1 + \rho = \frac{2 Z_{in}}{Z_{in} + Z_0} \tag{15}$$

The cubic line is terminated with a matching transmission line of characteristic impedance Z_L, then $a_2(\omega)=0$. In that case, relation (4) reveals that $S_{21}(\omega)$ is the ratio of b2(ω) to a1(ω) which are respectively equal to V(x=λ) and Vinc (incident wave) without forgetting the normalized impedance factor:

$$V_{inc} = \frac{V_{(x=0)}}{1 + \rho} \tag{16}$$

$$S_{21}(\omega) = \sqrt{\frac{Z_0}{Z_L}} \frac{V_{(x=\lambda)}}{V_{(x=0)}} (1+\rho) \quad (17)$$

Using the ratio p/q, $S_{21}(\omega)$ is directly expressed from a, b, ap and bp.

$$S_{21}(\omega) = \sqrt{\frac{Z_0}{Z_L}} (-2j) \frac{ap \cdot b - a \cdot bp}{-\frac{\omega}{v} b + j \cdot bp + j \cdot a + \frac{v}{\omega} \cdot ap} \quad (18)$$

At the input end, ρ is equal to the scattering reflection coefficient $S_{11}(\omega)$.

$$S_{11}(\omega) = \frac{-\frac{\omega}{v} b + j \cdot bp - j \cdot a - \frac{v}{\omega} \cdot ap}{-\frac{\omega}{v} b + j \cdot bp + j \cdot a + \frac{v}{\omega} \cdot ap} \quad (19)$$

Following the reciprocity principle, $S_{12}(\omega)$ equals $S_{21}(\omega)$. For the calculation of $S_{22}(\omega)$, the above theory is applied with an inversion of the impedance variation. New coefficients of the cubic polynomial are then computed :

$$Z_N 0 = Z_L = Z_0 + Z_1 \cdot \lambda + Z_2 \cdot \lambda^2 + Z_3 \cdot \lambda^3$$
$$Z_N 1 = -Z_1 - 2 \cdot Z_2 \cdot \lambda - 3 \cdot Z_3 \cdot \lambda^2$$
$$Z_N 2 = Z_2 + 3 \cdot Z_3 \cdot \lambda$$
$$Z_N 3 = Z_3$$

Before using the scattering parameters $S_{11}(\omega)$, $S_{21}(\omega)$, $S_{12}(\omega)$ and $S_{22}(\omega)$ in a global system, the reference impedance must be changed.

III-2 Analysis and optimization of the whole line

The scattering parameters of each line whose characteristic impedance is defined by a cubic polynomial must be characterized in the same reference system. Using changing impedance reference formulas, each line is defined between two reference lines such as $Z_{ref} = 50\ \Omega$. Moreover, to determine the global scattering parameters of the line defined by cubic spline interpolation, each scattering matrix of cubic polynomial line is transformed in chain matrix, then cascaded to be converted again in scattering parameters.

The scattering parameters of a transmission line whose characteristic impedance is described by a cubic spline interpolation are now exactly known. At this step, the theory to analyze non-uniform transmission lines is complete, but it has to be coupled with an optimization algorithm in order to connect the global shape of the line to the objective. The above theory suits well with an optimization method, because the shape of the line is described by a set of discrete points. Our goal here is an optimum filter response. $S_{11}(\omega)$ is the characteristic to achieve. The optimization method is based on the Levenberg-Marquardt and J. More algorithm [6] coupled with a random generation of the input discrete points Zd_j (for j varying from 1 to n) in order to pass over local minimums. A coefficient, k, which is a multiplier of the global line length is also optimized; k acts as a dilatation factor.

IV - Results

To validate by experiment the design theory, a stopband filter was optimized, designed and carried out. The center frequency of the filter is 3 GHz, with a 2 GHz bandwidth. The optimization goal was to reach -0.001 dB in the band for S_{11}(dB), and -30 dB out of the band. Fig. 2 shows the smooth impedance profile optimized and designed by the proposed method.

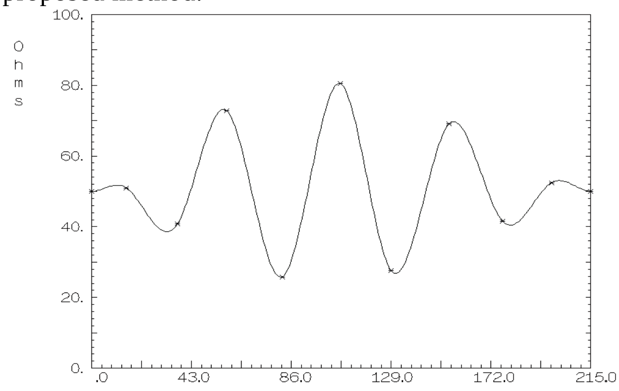

Fig. 2 - Impedance variation described by 10 discrete points.

The filter was made in microstrip technology. The substrate used was Isoclad GR6, the constructor gives a relative permittivity of 6 ± 0.15 and a thickness of 0.635 mm. The shape of this microstrip line is given in Fig. 3.

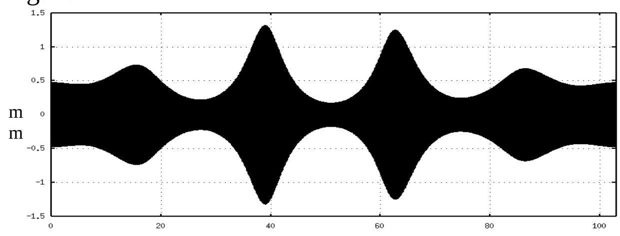

Fig. 3 - Shape of the microstrip filter (not to scale).

The comparison between the measured and simulated responses of $S_{11}(\omega)$ is displayed on Fig. 4. The measurements were done with a network analyser using an appropriate TRL calibration kit. This kit allowed us to verify the constructor permittivity value. An average of the measured value of this permittivity is about 6.4 in the considered bandwidth. Furthermore, an accurate evaluation of substrate thickness gives a value of 0.665 mm. The results presented on Fig. 4 take these measurements into account.

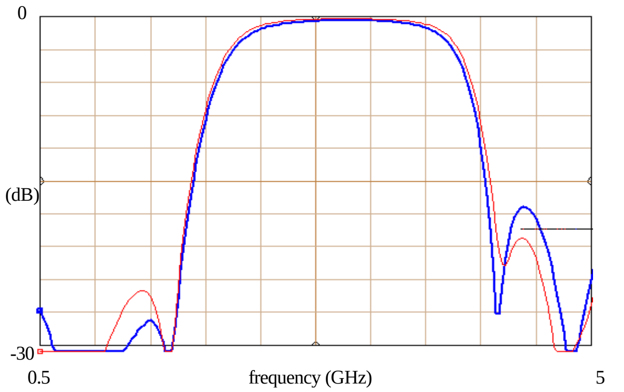

Fig 4 - Comparison between the simulated (A) and measured (B) frequency responses of S_{21}.

As shown in Fig. 4, the agreement between the theoretical and the measured response is very good. So, this realization validates the method used to analyze and optimize non-uniform transmission lines and thereby filters.

V - Discussion and perspectives

This new theory makes possible the design of other kinds of filters. For example lowpass, or bandpass filters with gaps could be developed under the condition of a TEM or quasi-TEM model. An example of a low-pass filter designed and optimized by our method is presented. The microstrip conductor profile is given for a permittivity of 9.6 and a substrate thickness of 0.635 mm. The simulated frequency response is also displayed.

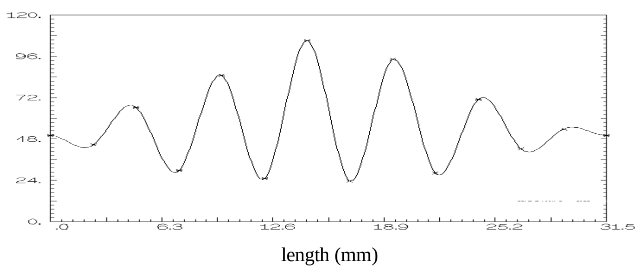

Fig. 5 - Impedance variation of the low-pass filter.

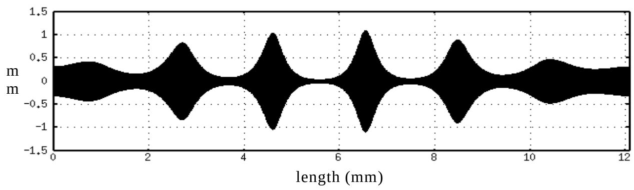

Fig. 6 - Shape of the microstrip low-pass filter.

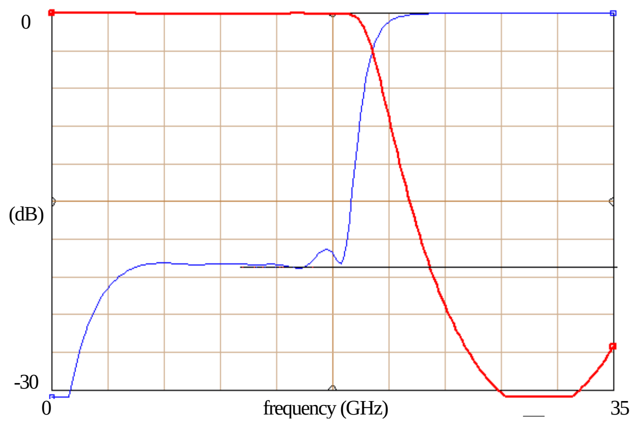

Fig. 7 - Simulated frequency response; S_{21} and S_{11} .

One of our next objectives is, for higher frequencies, to compare the performances of filters with smooth profile characterized by our model with other filters described by common techniques; i.e. equivalent distributed synthesis from Tchebyshev lumped filter. Filter responses could be compared up to 20 GHz to evaluate the influence of sharp discontinuities on radiation. An optimization of each distance between two discret points can also be achieved, in order to reject the raise back of S_{21} for low-pass filters.

VI - Conclusion

The validity of the theoretical response even over a wideband was established; then, a new method for microwave filter design with non-uniform transmission line is demonstrated conclusively. The method described hereabove presents distinct advantages over usual techniques. First, the structure shape has a greater potential (degrees of freedom for the optimization) than common lines. Secondly, the filter does not need to be described by a rational transfer function. Third, non-uniform lines avoid problems caused by sharp impedance discontinuities. Moreover, the analysis theory is useful to characterize different kinds of non-uniform transmission lines. Impedance matching lines for multi-stage amplifiers can also be designed by this technique and applied with the same iterative approach as the real frequency method [7].

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